UNIT – II

INTERPOLATION

Interpolation with Equal Intervals:

⊛ Newton's Forward Interpolation formula:

$$y(x) = y_p(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \dots$$

where $p = \frac{x - x_0}{h}$, $h \rightarrow$ (constant differences) in x-series Interval

⊛ Newton's Backward Interpolation Formula:

$$y(x) = y_p(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \dots$$

where $p = \frac{x - x_n}{h}$, $h \rightarrow$ Interval in x-series

Note:

① These formula is also called Newton's Gregory forward and Backward interpolation formula.

① Using Newton's interpolation formula find y(1.02) and y(1.35) from the following table:

x	1.0	1.1	1.2	1.3	1.4
y = f(x)	0.841	0.891	0.932	0.964	0.985

Soln.

Form the Difference Table

h	x (x_0)	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
	1.0	(y_0) 0.841				
0.1			0.050	Forward	value	
	1.1	0.891		-0.009		
0.1			0.041		0	
	1.2	0.932		-0.009		-0.002
0.1			0.032		-0.002	
	1.3	0.964		-0.011		
0.1			0.021			
	1.4	0.985 (y_n)			Backward values	

To find y(1.02):

Newton's forward interpolation formula

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

From Table, consider forward values

$y_0 = 0.841$, $x_0 = 1.0$,

$\Delta y_0 = 0.050$, $\Delta^2 y_0 = -0.009$, $\Delta^3 y_0 = 0$

$p = \dfrac{x - x_0}{h}$, $h = 0.1$

$x = 1.02$

$$\boxed{p = \frac{1.02 - 1.0}{0.1} = 0.2}$$

$\therefore y(1.02) = 0.841 + (0.2)(0.050)$

$+ \dfrac{(0.2)(0.2-1)}{2!}(-0.009) + 0$

$$\boxed{y(1.02) = 0.852}$$

To find $y(1.35)$:

Newton's Backward interpolation formula

$y_p(x) = y(x) = y_n + p\nabla y_n + \dfrac{p(p+1)}{2!}\nabla^2 y_n$

$+ \dfrac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \ldots$

From Table, consider Backward values

$y_n = 0.985$, $x_n = 1.4$, $h = 0.1$

$\nabla y_n = 0.021$, $\nabla^2 y_n = -0.011$, $x = 1.35$

$\nabla^3 y_n = -0.002$, $\nabla^4 y_n = -0.002$

$$p = \frac{x - x_n}{h} = \frac{1.35 - 1.4}{0.1} = -0.5$$

$$y(1.35) = 0.985 + (-0.5)(0.021) + \frac{(-0.5)(-0.5+1)}{2!}(-0.011) + \frac{(-0.5)(-0.5+1)(-0.5+2)}{3!}(-0.002) + \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)}{4!}(-0.002)$$

$$= 0.985 - 0.0105 + 0.001375 + 0.000125 + 0.0001$$

$$\boxed{y(1.35) = 0.9761}$$

(2) Estimate $\exp(1.85)$ from the following table using Newton's forward interpolation formula.

x :	~~1.7~~	1.8	1.9	2.0	2.1	2.2	2.3
$y = e^x$:	~~5.474~~	6.050	6.686	7.389	8.166	9.025	9.974

Sol: Form the Difference Table:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.8	6.050					
		0.636				
1.9	6.686		0.067			
		0.703		0.007		
2.0	7.389		0.074		0.001	
		0.777		0.008		-0.001
2.1	8.166		0.082		0	
		0.859		0.008		
2.2	9.025		0.090			
		0.949				
2.3	9.974					

From Table

$x_0 = 1.8$ $y_0 = 6.050$, $x = 1.85$, $h = 0.1$

$\Delta y_0 = 0.636$ $\Delta^2 y_0 = 0.067$ $\Delta^3 y_0 = 0.007$

$\Delta^4 y_0 = 0.001$, $\Delta^5 y_0 = -0.001$

By Newton's forward interpolation formula

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \cdots$$

$$p = \frac{x - x_0}{h} = \frac{1.85 - 1.8}{0.1} = 0.5$$

$$y(1.85) = 6.050 + \frac{(0.5)(0.636)}{1!} + \frac{(0.5)(-0.5)}{2!}(0.067)$$
$$+ \frac{(0.5)(-0.5)(-1.5)}{3!}(0.007) + \cdots$$

$$\boxed{y(1.85) = 6.3601}$$

3. From the following data, using Newton's Backward interpolation formula, compute Log(58.75).

x:	40	45	50	55	60	65
Log x:	1.6021	1.6532	1.6990	1.7403	1.7781	1.8125

Form the Difference Table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	1.6021				
		0.0511			
45	1.6532		-0.0053		
		0.0458		0.0008	
50	1.6990		-0.0045		0.0002
		0.0413		0.0010	
55	1.7403		-0.0035		
		0.0378			
60	1.7781				
65					

Here $x_n = 60$, $h = 5$, $y_n = 1.7781$

Let $x = 58.75$, $p = \frac{x - x_n}{h} = \frac{58.75 - 60}{5}$

$= -0.25$

Backward interpolation formula is

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \dots$$

from Table

$\nabla y_n = 0.0378$ $\quad \nabla^2 y_n = -0.0035$

$\nabla^3 y_n = 0.0010$ $\quad \nabla^4 y_n = 0.0002$

$$y(58.75) = 1.7781 - (0.25)(0.0378) + \frac{(-0.25)(0.75)}{2}(-0.0035) + \frac{(-0.25)(0.75)(1.75)}{6}(0.00$$

$$+ \frac{(-0.25)(0.75)(1.75)(2.75)}{24}(0.0002)$$

$$= 1.7781 - 0.00945 + 0.00033 - 0.00005 - 0.000$$

$$\boxed{y(58.75) = 1.7689}$$

(4) Find the cubic polynomial which takes the value $y(0) = 1$, $y(1) = 0$, $y(2) = 1$, and $y(3) = 10$. Hence obtain $y(4)$.

Sol

Difference Table:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
0	1			
		-1		
1	0		2	
				6
2	1	1		
			8	
3	10	9		

By Newton's forward interpolation formula

$$y(x) = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \dots$$

$$x_0 = 0, \quad h = 1$$

$$p = \frac{x - x_0}{h} = x$$

$$\Delta y_0 = -1, \quad \Delta^2 y_0 = 2 \quad \Delta^3 y_0 = 6$$

$$y(x) = 1 + x(x-1) + \frac{x(x-1)}{2}(2) + \frac{x(x-1)(x-2)}{6}(6)$$

$$= 1 - x + x^2 - x + x^3 - 3x^2 + 2x$$

$$\boxed{y(x) = x^3 - 2x^2 + 1}$$ is the cubic polynomial

Also find $y(4)$.

$$y(4) = (4)^3 - 2(4)^2 + 1$$

$$\boxed{y(4) = 64 + 1 - 32 = 33}$$

H/W

① From the following table values of x and $f(x)$, determine $f(0.23)$ & $f(0.29)$

x :	0.20	0.22	0.24	0.26	0.28	0.30
$f(x)$:	1.6596	1.6698	1.6804	1.6912	1.7024	1.7139

Ans: $f(0.23) = 1.6751$

$f(0.29) = 1.7081$

(2) Using Newton's Backward interpolation formula, find y, when $x = 27$ from the following data.

x :	10	15	20	25	30
y :	35.4	32.2	29.1	26.0	23.1

Ans: $y(27) \simeq 24.7947$

(3) Find the polynomial which passes through the points (7, 3) (8, 1) (9, 1) and (10, 9) using Newton's interpolation formula.

Ans: $y(x) = x^3 - 23x^2 + 174x - 431$

———— x ————